

### SUPERVISOR'S DECLARATION

We hereby declare that we have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Doctor of Philosophy



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SPATIAL MODELLING OF EXTREME  
RAINFALL AMOUNT IN KELANTAN

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## ABSTRAK

Kajian terhadap model jumlah hujan lampau adalah sangat penting untuk memberikan anggaran yang tepat mengenai proses hujan lampau dan dapat menjana siri data sintetik hujan lampau yang panjang dalam kes-kes di mana data adalah terhad. Kajian terhadap sifat peristiwa ekstrim agak mencabar dan memerlukan pencirian terhadap ekor taburan. Peristiwa ekstrim memerlukan pemodelan data ekstrim berasaskan kawasan bagi mengkaji dan mencirikan kelakuan serantau dalam kawasan kajian. Maka, kajian ini memberi tumpuan kepada model univariat dan multivariat hujan lampau di Kelantan, Malaysia dengan mengambil kira kesan berasaskan kawasan. Pertama, pengecaman siri hujan lampau dapat dicapai menggunakan dua kaedah, iaitu kaedah maksimum tahunan dan kaedah puncak atas ambang. Permasalahan timbul bagi memilih kaedah yang sesuai untuk mengekstrak data hujan lampau. Bagi kebanyakan kes, para penyelidik akan memilih menggunakan kaedah siri maksimum tahunan atau siri puncak atas ambang. Maka, bagi kes data hujan univariat di mana data dikumpulkan mengikut lokasi kawasan, kami mencadangkan prosedur baru bagi kaedah analisis kekerapan serantau dengan menggunakan kaedah siri maksimum tahunan dan siri puncak atas ambang untuk mengekstrak data hujan lampau. Kedua, kami mempertimbangkan kes analisis multivariat berasaskan kawasan yang mana struktur pergantungan antara stesen hujan diambil kira dengan menggunakan model *max-stable*. Amalan semasa bagi proses *max-stable* ialah dengan memodelkan pergantungan ekstrim berasaskan kawasan menggunakan puncak atas ambang yang tetap dan model marginal yang tetap. Dalam kajian ini, kami mencadangkan prosedur baru untuk model *max-stable* dengan mengambil kira nilai ambang yang berasaskan kawasan dan taburan marginal yang bergantung kepada kovariat yang sesuai bagi kawasan kajian yang besar. Kami juga menggunakan model *max-stable* dengan mencirikan stesen hujan dengan sifat yang sama menggunakan kaedah analisis kekerapan serantau. Tiga model dari *max-stable* digunakan iaitu *Smith*, *Schlather* dan *Brown-Resnick* model. Kami juga mempertimbangkan kes yang mana struktur pergantungan tidak menumpu kepada had *max-stable* dengan menggunakan model ketakbersandaran asimptot. Ketiga, kami mempertimbangkan kaedah lain untuk menggabungkan pergantungan berasaskan kawasan bagi hujan ekstrim menggunakan model *copula*. *Multivariate skew-t copula* dibina untuk permodelan data hujan lampau berasaskan kawasan. Kami telah melanjutkan kes bivariat kepada kes trivariate *skew-t copula* di mana tiga stesen hujan akan dipertimbangkan. Keempat, kami mengambil kira penjanaan data hujan sintetik yang mempunyai ciri-ciri yang sama dengan data cerapan. Oleh itu, dalam kajian ini, kami mencadangkan penjanaan data hujan sintetik yang boleh melengkapkan ketiadaan data hujan menggunakan model terbaik berasaskan kawasan iaitu kaedah analisis kekerapan serantau, *max-stable* dan *copula*. Akhir sekali, kami menerangkan dan membincangkan kekuatan model hujan lampau yang telah dicadangkan dalam kajian ini. Kami membangunkan profil hujan berasaskan kawasan bagi Kelantan berdasarkan ciri-ciri kawasan. Sebagai ilustrasi, kajian menggunakan data hujan bagi kawasan Kelantan. Keputusan menunjukkan bahawa kaedah analisis kekerapan serantau, *max-stable* dan *copula* berjaya digunakan bagi kawasan kajian. Apabila membandingkan semua kaedah ini, tidak ada corak konsisten dalam pelaksanaan setiap kaedah. Oleh itu, tiada kesimpulan yang pasti dapat dibuat pada kaedah yang terbaik atau yang paling sesuai. Pilihan tempat model hujan yang terbaik adalah subjektif bergantung kepada tujuan kajian itu sendiri. Kajian yang berlainan menggunakan kaedah yang berbeza untuk anggaran hujan, yang menjadikan ia sukar untuk membuat kesimpulan umum.

Walau bagaimanapun, berdasarkan kepada keputusan yang diperolehi, ia menunjukkan bahawa kaedah analisis kekerapan serantau, *max-stable* dan *copula* boleh digunakan untuk model data hujan lampau di Kelantan apabila pergantungan berasaskan kawasan adalah satu keutamaan. Walau bagaimanapun, semua model yang dicadangkan iaitu analisis kekerapan serantau, *max-stable* dan *copula* menunjukkan prestasi yang baik dan dapat digunakan pada set data sebenar. Sumbangan signifikan kajian ini mengenai pergantungan berasaskan kawasan dalam pemodelan hujan lampau adalah penting untuk mengurangkan ketidakpastian anggaran titik bagi indeks taburan ekor. Selain itu, kajian ini adalah penting kerana ia boleh menyediakan satu anggaran ciri-ciri serantau hujan lampau di Kelantan, Malaysia yang berguna dalam anggaran banjir untuk menyokong pengurusan risiko banjir dan juga untuk reka bentuk kejuruteraan.



## ABSTRACT

The study on extreme rainfall modelling is very important to estimate the extreme rainfall process accurately and to generate a long series of synthetic extreme rainfall data in cases where data is limited. Assessing the behaviour of extreme events is quite challenging and requires the characterisation of the tail distribution. The extreme events require the modelling of spatial extreme data in order to analyse and characterise the regional behaviour in a study region. Hence, this study focuses on the univariate and multivariate spatial modelling of extreme rainfall data with application in Kelantan, Malaysia. Firstly, the identification of extreme rainfall data can be achieved using two methods, namely block maximum (BM) and peak over threshold (POT) methods. The difficulty arises in choosing the appropriate method to extract the extreme rainfall data. In most cases, the researcher will choose either POT or BM method in extracting extreme rainfall. Hence, for the case where univariate rainfall data are collected at the spatial locations, we proposed a new procedure of regional frequency analysis (RFA) by considering the BM and POT methods to extract extreme rainfall data. Secondly, we considered the case of spatial multivariate analysis where the dependence structures between the rainfall stations were taken into account using the max-stable model. A current practice on max-stable process is to model the dependence for spatial extremes using some constant thresholds and constant marginal models. In this study, we proposed a new procedure for max-stable model by considering different spatial threshold with marginal distribution that depends on covariates that suits well for large study regions. We also applied the max-stable model by regionalising the stations with similar characteristics using the RFA method. Three models from max-stable were considered namely Smith, Schlather and Brown-Resnick models. We also considered the case when the dependence structure does not converge to the max-stable limit using the inverted max-stable model. Thirdly, we considered another method to incorporate the spatial dependence of extreme rainfall using the copula model. The multivariate skew- $t$  copula was constructed to model the spatial extreme of rainfall data. We have extended the bivariate case to the trivariate case of skew- $t$  copula where three rainfall stations were considered. Fourthly, we considered the generation of synthetic extreme rainfall data that has similar characteristics to the observed data. Thus, in this study, we have proposed the generation of synthetic rainfall data that can complement the unavailability of the observed rainfall data using the best spatial model from the RFA, max-stable and copula methods. Lastly, we described and discussed the strengths of the extreme rainfall models that have been proposed in this study. We developed the spatial rainfall profile for Kelantan based on its spatial characteristics. The applications of all the models were illustrated using rainfall amount data of Kelantan. The results of the study shows that the RFA, max-stable and copula methods were successfully applied to the study region. By comparing the performance of all models used, it was found that there was no consistent pattern in the performance of each method and thus, no definite conclusion could be drawn on which method is the best or most appropriate. The choice of the best rainfall model is subjective depending on the purpose of the study itself. Different studies used different methods for rainfall estimation, which makes it difficult to draw a general conclusion. However, based on the results obtained, it was shown that the RFA, max-stable and copula methods can be used to model the extreme rainfall data in Kelantan when the spatial dependence is of the main concern. Nevertheless, all the proposed model that are RFA, copula and max-stable model perform well and able to demonstrate its usefulness on real data sets. The significant contribution of the study on

spatial dependence in extreme rainfall modelling is important to reduce the uncertainties of the point estimates for the tail index. Also, the study is important as it provides an estimate of regional characteristics of extreme rainfall in Kelantan, Malaysia that is useful in flood estimation to support flood risk management as well as for engineering design.

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## LIST OF SYMBOLS

$D_i$	Discordancy measure
$\bar{u}$	Unweighted group average
$\hat{\tau}$	Sample $L$ -coefficient of variation ( $L$ -CV)
$\hat{\tau}_3$	Sample $L$ -skewness
$\hat{\tau}_4$	Sample $L$ -kurtosis
$\tau$	Population $L$ -coefficient of variation ( $L$ -CV)
$\tau_3$	Population $L$ -skewness
$\tau_4$	Population $L$ -kurtosis
$V_s$	The weighted standard deviation of the simulated values
$H_j$	Heterogeneity measure
$\beta_4$	Bias correction
$N_{sim}$	Number of the simulation
$\lambda_r$	$r^{th}$ population of $L$ -moment
$Q(F)$	Quantile estimate
$g_1$	Index rainfall value
$R_q(F)$	The root mean square error
$M$	Number of repetition of bias
$F(x)$	Cumulative distribution function
G1	Cluster 1
G2	Cluster 2
G3	Cluster 3
a.s.l	Above sea level
$Z(s)$	The stochastic process
$\theta_D$	Extremal coefficient
$V_e$	Exponent measure
$M_n$	Highest rainfall amount over $n$
$u$	Threshold
$X$	Random variable

$\mu$	Location parameter
$\alpha$	Shape parameter
$\beta$	Scale parameter
$M_p$	Moment functional of order $p$
$Q_x(a)$	Quantile function
$\theta$	Unknown parameter
$T_r$	Return period
$\hat{\rho}$	Spearman's correlation coefficient
$N$	Number of rainfall station
$s$	Rainfall station
$S$	Study region
$t$	Time
$e(u)$	Mean residual life plot
$I$	Indicator function
$Q_i$	Empirical quantile
$g$	Number of lags
$d(C_i, C_j)$	Distance between Cluster $C_i$ and $C_j$
$d(x, y)$	Distance between rainfall station $x$ and $y$
$\xi_i$	Points of the Poisson process II
$W_i(s)$	Independent replicates of the non-negative random process
$\Sigma$	Covariance matrix
$\ h\ $	Euclidean distance
$\sigma$	Nugget parameter
$\lambda$	Range parameter
$\kappa$	Smoothness parameter
$e_i$	Independent copies of a zero mean Gaussian process
$\gamma(h)$	Semivariogram
$\Phi$	Standard normal cumulative distribution function
$\psi$	Probability distribution function
$\mathcal{L}(z)$	Slowly varying function

$\eta$	Coefficient of tail dependence
$w_{ij}$	Non-negative weights
$C$	Copula
$\mathbf{L}$	Lower-triangular matrix
$\nu$	Degrees of freedom
$\boldsymbol{\mu}$	Mean vector
$L_n$	Normalising constant
$H_\lambda$	Modified Bessel function of third kind
$\sigma_4$	The standard deviation of weighted regional average
$M_i$	Length of rainfall data at rainfall station $i$
$t^R$	Weighted regional average
$\mu_t$	Theoretical mean
$\sigma_t^2$	Theoretical variance
$\gamma_t$	Theoretical skewness
$\kappa_t$	Theoretical kurtosis
$\lambda_c$	Log-composite likelihood
$D_n$	Maximum of the absolute difference between the cumulative distribution function of the observed and generated data
$L_k$	Maximised value of the likelihood function for the fitted model
$k$	Number of parameter
tr	Trace
$\mathbf{H}$	Hessian matrix
$\mathbf{J}$	Covariance matrix of the score function
$\mathbf{P}$	Correlation matrix
$D_\theta$	Composite score function
$n$	Number of extreme rainfall data
$\hat{\rho}_m$	Sample autocorrelation at lag $m$
$r_i$	Centroid of cluster $C_i$

$r_{ij}$	Centroid of cluster $C_{ij}$
$h_i$	Inverse-link function
$t_1$	Covariate function of latitude
$t_2$	Covariate function of longitude
$l_r$	Sample $L$ -moment
$L_c$	Composite likelihood
$\mathbf{H}$	Estimated Hessian matrix
$\mathbf{J}^{-1}$	Estimated covariance matrix
$\rho$	Correlation
$F$	Distribution function
$q$	Quantile function
$R^R$	Regional average relative root mean square error
$\mathfrak{D}$	Domain
$H$	Number of exceedances in a study region, $S$
$A$	Cholesky decomposition of $P$
$\gamma$	Skewness parameter vector
$\lambda_p(x)$	Intensity
$\lambda_p$	Parameter of Poisson distribution



## LIST OF ABBREVIATIONS

AA	Arithmetic Average
AHC	Agglomerative hierarchical cluster
AIC	Akaike information criterion
BF	Bayes factor
BIC	Bayesian information criterion
BM	Block maximum
BM1	Block maximum of 1-day
BM3	Block maximum of 3-day
BM5	Block maximum of 5-day
BM10	Block maximum of 10-day
CC	Correlation coefficient
cdf	Cumulative distribution function
CLIC	Composite likelihood information criterion
CV	The coefficient of variation
DIC	Deviance information criterion
DID	Department of Irrigation and Drainage
EVT	Extreme value theory
EVD	Extreme value distribution
GEV	Generalised extreme value
GP	Generalised pareto
GLO	Generalised logistics
GNO	Generalised normal
ID	Inverse Distance
i.i.d	Independent and identically distributed
MLE	Maximum likelihood estimation
MCLE	Maximum composite likelihood estimation
MK	Mann Kendall
MAP	Mean annual precipitation
MSE	Mean squared error
MI	Multiple imputation
NR	Normal ratio



pdf	Probability distribution function
POT	Peak over threshold
POT95	Peak over threshold series of 95% quantile estimate
POT96	Peak over threshold series of 96% quantile estimate
POT97	Peak over threshold series of 97% quantile estimate
PE3	Pearson Type III
PWM	Probability weighted moment
QQ	Quantile quantile
RFA	Regional frequency analysis
RMSE	Root mean square error
S-Index	Similarity index
SSE	Sum of squares error
SST	Sum of squares total
vs	Versus
WAK	Wakeby

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